Properties of State Transition Matrix

$$\phi(t) = e^{At} = \text{State Transition Matrix}$$

(i) $\phi(0) = e^{A \times 0} = I = \text{Identity Matrix}$

(ii) $\phi(t) = e^{At} = (e^{-At})^{-1} = [\phi(-t)]^{-1}$

i.e. $\phi^{-1}(t) = \phi(-t)$

(iii) $\phi(t_1 + t_2) = e^{A(t_1+t_2)} = e^{At_1} \cdot e^{At_2}$

$= \phi(t_1) \cdot \phi(t_2) = \phi(t_2) \cdot \phi(t_1)$

(iv) $e^{A(t+s)} = e^{At} \cdot e^{As}$

(v) $e^{(A+B)t} = e^{At} \cdot e^{Bt}$ only if $AB = BA$

(vi) $[\phi(t)]^n = [e^{At}]^n = e^{Ant} = \phi(nt)$

(vii) $\phi(t_2 - t_1) * \phi(t_1 - t_0) = \phi(t_2 - t_0)$

(viii) $\phi(t)$ is a non-singular matrix for all finite values of t.